Activities/ Resources for Outcome #3

Fractions: Notes and Discovery

Writing Fractions in Simplest Form

<u>Definition:</u> A quotient of two numbers is called a **fraction**. In the fraction, the top number is called the **numerator** and the bottom number is called the **denominator**.

Ex: $\dfrac{3}{4}$ is a fraction where 3 is the numerator, 4 is the denominator

To simplify fractions, we can **factor** the denominator. **Factoring** means to write a number as a product.

Ex: $2 \cdot 5 = 10$ OR $1 \cdot 10 = 10$

          ↑   ↑

    factors   product

<u>Definition:</u> A fraction is said to be **simplified** or in **lowest terms** when the numerator and denominator have no common factors other than 1.

To make it easier to simplify a fraction, the numerator and denominator should be written as products of **prime numbers**.

<u>Definition:</u> A **prime number** is a natural number, other than 1, whose only factors are 1 and itself.

        Ex: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc

<u>Definition:</u> A **composite number** is a natural number that is not prime. Every composite number can be written as a unique product of primes.

        Ex: $24 = 4 \cdot 6$

             ↓

           $= 2 \cdot 2 \cdot 2 \cdot 3$

Write 60 as a product of primes:

        $60 =$

<u>Fundamental Principle of Fractions:</u>   If $\dfrac{a}{b}$ is a fraction and c is a nonzero real

number, then $\dfrac{a \cdot c}{b \cdot c} = \dfrac{a}{b}$.

Why is this true? (Try to think of an example or two.)

Writing fractions in lowest terms:

Ex: a.) $\dfrac{30}{36} = \dfrac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 3} = \dfrac{5}{6}$

       b.) $\dfrac{13}{24} = \dfrac{13}{2 \cdot 2 \cdot 2 \cdot 3} = \dfrac{13}{24}$

Write the following fraction in lowest terms:

Ex: $\dfrac{20}{35}$

Multiplying and Dividing Fractions

<u>Multiplying Fractions:</u> $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{a \cdot c}{b \cdot d}$ if $b \neq 0$ and $d \neq 0$

Multiplying fractions and writing in lowest terms:

Ex: a.) $\dfrac{3}{4} \cdot \dfrac{2}{11} = \dfrac{3 \cdot 2}{4 \cdot 11} = \dfrac{3 \cdot 2}{2 \cdot 2 \cdot 11} = \dfrac{3}{22}$

       b.) $\dfrac{5}{8} \cdot \dfrac{4}{7} = \dfrac{5 \cdot 4}{8 \cdot 7} = \dfrac{5 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 7} = \dfrac{5}{14}$

Multiply the following fractions and write the product in lowest terms:

Ex: $\dfrac{2}{7} \cdot \dfrac{3}{10} =$

<u>Definition:</u>   Two fractions are **reciprocals** of each other if their product equals 1.

Ex: $\dfrac{3}{4}$ and $\dfrac{4}{3}$ are reciprocals because $\dfrac{3}{4} \cdot \dfrac{4}{3} = 1$

<u>Dividing Fractions:</u> $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$ if $b \neq 0$, $c \neq 0$, and $d \neq 0$

Dividing fractions and writing in lowest terms:

Ex: a.) $\dfrac{2}{5} \div \dfrac{5}{9} = \dfrac{2}{5} \cdot \dfrac{9}{5} = \dfrac{2 \cdot 9}{5 \cdot 5} = \dfrac{18}{25}$

b.) $\dfrac{3}{4} \div \dfrac{7}{12} = \dfrac{3}{4} \cdot \dfrac{12}{7} = \dfrac{3 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 7} = \dfrac{9}{7} = 1\dfrac{2}{7}$

Divide the following fractions and write the quotient in lowest terms:

Ex: a.) $\dfrac{2}{9} \div \dfrac{3}{4} =$

b.) $\dfrac{5}{8} \div 10 =$

Adding and Subtracting Fractions

<u>Adding and Subtracting Fractions with the Same Denominator:</u>

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \qquad \text{if } b \neq 0$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b} \qquad \text{if } b \neq 0$$

Adding and subtracting fractions and writing in lowest terms:

Ex: a.) $\dfrac{1}{5} + \dfrac{3}{5} = \dfrac{1+3}{5} = \dfrac{4}{5}$

b.) $\dfrac{7}{8} - \dfrac{5}{8} = \dfrac{7-5}{8} = \dfrac{2}{8} = \dfrac{2}{2 \cdot 2 \cdot 2} = \dfrac{1}{4}$

Add or subtract the following fractions and write the answer in lowest terms:

Ex:　　a.)　　$\dfrac{2}{11} + \dfrac{5}{11} =$

　　　　b.)　　$\dfrac{13}{10} - \dfrac{3}{10} =$

Adding and Subtracting Fractions with Different Denominators:

In order to add or subtract fractions with different denominators, you must find the **least common denominator (LCD)** amongst the fractions. Once the LCD is found, write each fraction as an **equivalent fraction** with the LCD as the new denominator.

Ex:　　a.)　　$\dfrac{1}{6} + \dfrac{2}{9} =$

　　　　Since 6 and 9 are obviously not the same, you must find their LCD. What is the smallest multiple of 6 AND 9?
　　　　[Hint: It will be the first multiple of the larger number (9) that is a multiple of the smaller number (6).]

　　　　Multiples of 9:　　9, 18, 27, 36, …

　　　　18 is the first number that is also a multiple of 6 so 18 is the LCD.

　　　　Now, change each fraction into an equivalent fraction with 18 as the denominator…

　　　　What number multiplied by 6 will give you 18? 3!

　　　　$\dfrac{1}{6} \cdot \dfrac{3}{3} = \dfrac{3}{18}$

　　　　What number multiplied by 9 will give you 18? 2!

　　　　$\dfrac{2}{9} \cdot \dfrac{2}{2} = \dfrac{4}{18}$

　　　　Now you are ready to add the two fractions together.

　　　　$\dfrac{1}{6} + \dfrac{2}{9} = \dfrac{3}{18} + \dfrac{4}{18} = \dfrac{7}{18}$

b.) $\quad 4\dfrac{2}{3} - 2\dfrac{5}{6} = \dfrac{14}{3} - \dfrac{17}{6} = \dfrac{28}{6} - \dfrac{17}{6} = \dfrac{11}{6} = 1\dfrac{5}{6}$

Add or subtract the following fractions and write the difference in lowest terms:

Ex: a.) $\quad \dfrac{3}{8} + \dfrac{1}{20} =$

  b.) $\quad 18\dfrac{1}{4} - 6\dfrac{2}{3} =$

Add.

1) $^{16}/_{19}$ + $^{19}/_{45}$

2) $^{1}/_{1}$ + $^{0}/_{39}$

3) $^{13}/_{49}$ + $^{9}/_{30}$

4) $^{37}/_{42}$ + $^{49}/_{49}$

5) $^{25}/_{39}$ + $^{9}/_{40}$

6) $^{3}/_{21}$ + $^{8}/_{43}$

7) $^{7}/_{9}$ + $^{19}/_{32}$

8) $^{4}/_{16}$ + $^{44}/_{48}$

9) $^{29}/_{37}$ + $^{1}/_{16}$

10) $^{28}/_{47}$ + $^{5}/_{5}$

Subtract.

1) $^3/_{32} - {}^0/_{27}$

2) $^{21}/_{46} - {}^{43}/_{47}$

3) $^{24}/_{46} - {}^{15}/_{22}$

4) $^{22}/_{45} - {}^{11}/_{38}$

5) $^{26}/_{44} - {}^{32}/_{37}$

6) $^{21}/_{30} - {}^{14}/_{46}$

7) $^{16}/_{33} - {}^3/_{38}$

8) $^{29}/_{46} - {}^4/_{16}$

9) $^{12}/_{40} - {}^{13}/_{25}$

10) $^{36}/_{38} - {}^{17}/_{39}$

Multiply.

1) $\frac{9}{41} \times \frac{20}{24}$

2) $\frac{1}{30} \times \frac{26}{46}$

3) $\frac{22}{46} \times \frac{27}{38}$

4) $\frac{3}{9} \times \frac{13}{26}$

5) $\frac{23}{33} \times \frac{11}{16}$

6) $\frac{25}{34} \times \frac{8}{12}$

7) $\frac{8}{24} \times \frac{19}{38}$

8) $\frac{2}{3} \times \frac{9}{17}$

9) $\frac{23}{41} \times \frac{7}{29}$

10) $\frac{16}{39} \times \frac{1}{45}$

Divide.

1) $^0/_{21} \div {}^{12}/_{22}$

2) $^{21}/_{34} \div {}^{24}/_{27}$

3) $^{20}/_{23} \div {}^{22}/_{48}$

4) $^{23}/_{45} \div {}^{10}/_{45}$

5) $^{35}/_{44} \div {}^{30}/_{36}$

6) $^{24}/_{45} \div {}^{15}/_{39}$

7) $^{13}/_{43} \div {}^{42}/_{48}$

8) $^4/_{42} \div {}^{22}/_{37}$

9) $^2/_{13} \div {}^{40}/_{46}$

10) $^{30}/_{47} \div {}^5/_{49}$