Activities/ Resources For Unit VI: Algebra and Geometry I

NEGATIVE & POSITIVE NUMBERS

Adding Real Numbers

Positive + Positive = Positive	5 + 3 = 8
Negative + Negative = Negative	(- 6) + (- 3) = - 9
	(- 6) + 3 = -3
Sum of a negative and a positive number	9 + (-12) = - 3
Keep the sign of the larger number and subtract	(- 5) + 7 = 2
	4 + (-2) = 2.

Subtracting Real Numbers:

Negative - Positive = Negative (same as adding two negative numbers)	(- 8) - 3 = -8 + (-3) = -11
Positive - Negative = Positive + Positive = Positive	4 - (-3) = 4 + 3 = 7
Negative - Negative = Negative + Positive =	(-7) - (-5) = (-7) + 5 = -2
Keep the sign of the larger number and subtract	(-5) - (-7) = (-5) + 7 = 2

Multiplying Real Numbers:

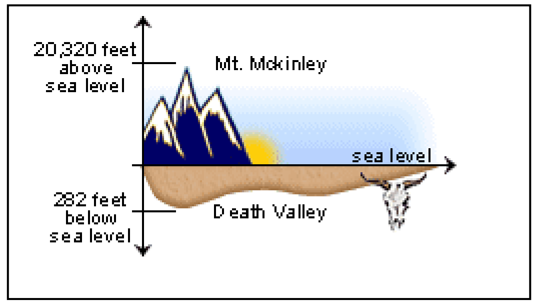
Positive x Positive = Positive	3 x 4 = 12
Negative x Negative = Positive	(-3) x (-5) = 15
Negative x Positive = Negative	(-3) x 2 = -6
Positive x Negative = Negative	3 x (-2) = -6

Dividing Real Numbers:

Positive ÷ Positive = Positive	18 ÷ 3 = 6
Negative ÷ Negative = Positive	(-18) ÷ (-3) = 6
Negative ÷ Positive = Negative	(-18) ÷ 3 = -6
Positive ÷ Negative = Negative	18 ÷ (-3) = -6.

Integers

Problem: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the distance from the top of Mt. McKinley to the bottom of Death Valley?

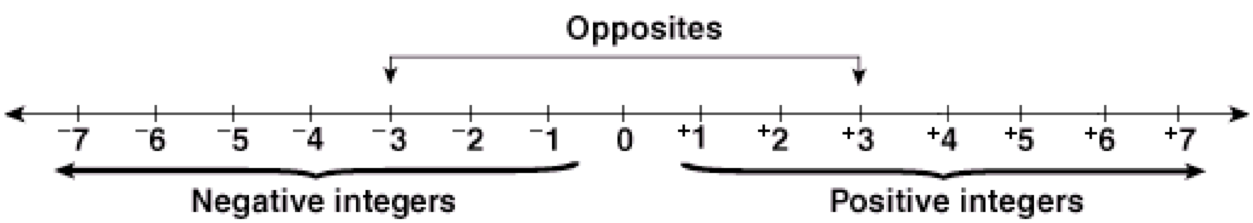


Solution: The distance from the top of Mt. McKinley to sea level is 20,320 feet and the distance from sea level to the bottom of Death Valley is 282 feet. The total distance is the sum of 20,320 and 282, which is 20,602 feet.

The problem above uses the notion of opposites: Above sea level is the opposite of below sea level. Here are some more examples of opposites:

top, bottom increase, decrease forward, backward positive, negative

We could solve the problem above using integers. **Integers are the set of whole numbers and their opposites.** The number line can be used to represent the set of integers. Look carefully at the number line below and the definitions that follow.

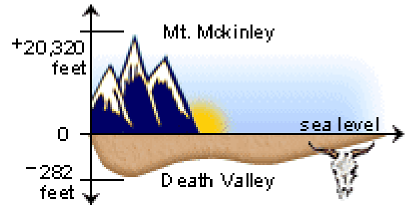


Definitions

- The number line goes on forever in both directions. This is indicated by the arrows.
- Whole numbers greater than zero are called **positive integers**. These numbers are to the right of zero on the number line.
- Whole numbers less than zero are called **negative integers**. These numbers are to the left of zero on the number line.
- The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive ($^{+}$) or negative ($^{-}$), except zero, which has no sign.
- Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. One will have a positive sign, the other a negative sign. In the number line above, $^{+}3$ and $^{-}3$ are labeled as opposites.

Let's revisit the problem from the top of this page using integers to solve it.

Problem: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the distance from the top of Mt. McKinley to the bottom of Death Valley?



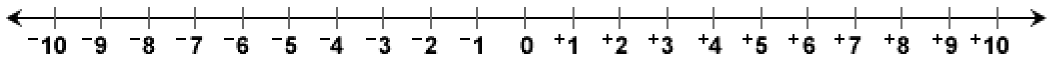
Solution: We can represent each elevation as an integer:

Elevation	**Integer**
20,320 feet above sea level	$^{+}20{,}320$
sea level	0
282 feet below sea level	$^{-}282$

The distance from the top of Mt. McKinley to the bottom of Death Valley is the same as the distance from $^{+}20{,}320$ to $^{-}282$ on the number line. We add the distance from $^{+}20{,}320$ to 0, and the distance from 0 to $^{-}282$, for a total of 20,602 feet.

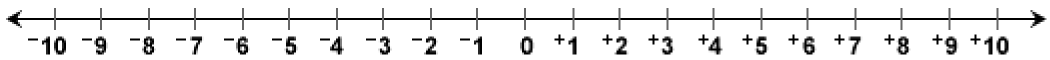
Example 1: Write an integer to represent each situation:

10 degrees above zero	$^{+}10$
a loss of 16 dollars	$^{-}16$
a gain of 5 points	$^{+}5$
8 steps backward	$^{-}8$



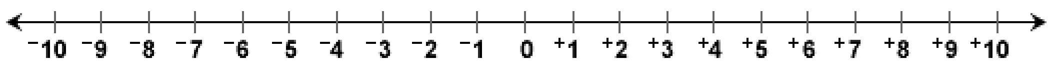
Example 2: Name the opposite of each integer.

$^{-}12$	$^{+}12$
$^{+}21$	$^{-}21$
$^{-}17$	$^{+}17$
$^{+}9$	$^{-}9$



Example 3: Name 4 real life situations in which integers can be used.

Spending and earning money.
Rising and falling temperatures.
Stock market gains and losses.
Gaining and losing yards in a football game.

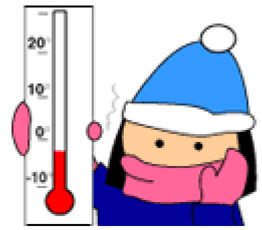


Note: A positive integer does not have to have a $^{+}$ sign in it. For example, $^{+}3$ and 3 are interchangeable.

Summary: Integers are the set of whole numbers and their opposites. Whole numbers greater than zero are called positive integers; whole numbers less than zero are called negative integers. The integer zero is neither positive nor negative, and has no sign. Two integers are opposites if they are each the same distance away from zero, but on opposite sides of the number line. A positive integer may be written with or without a sign.

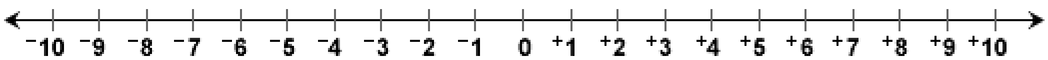
Subtraction of Integers

Problem: The temperature in Anchorage, Alaska was 8°F in the morning and dropped to $^{-}5$°F in the evening. What is the difference between these temperatures?

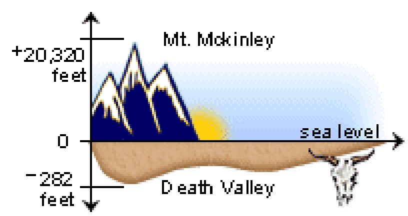


Solution: We can solve this problem using integers. Using the number line below, the distance from $^{+}8$ to 0 is 8, and the distance from 0 to $^{-}5$ is 5, for a total of 13.

$^{+}8$ - $^{-}5$ = $^{+}13$. The difference is 13 degrees.



Problem: The highest elevation in North America is Mt. McKinley, which is 20,320 feet above sea level. The lowest elevation is Death Valley, which is 282 feet below sea level. What is the difference between these two elevations?



Solution: When solving problems with large integers, it is not always practical to rely on the number line. Using integer arithmetic this problem becomes: $^{+}20,320$ - $^{-}282$ = ?

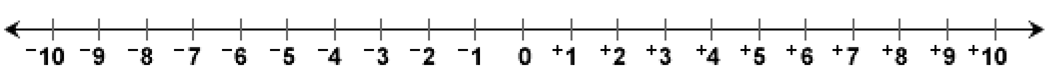
We need a rule for subtracting integers in order to solve this problem.

Rule: To subtract an integer, add its opposite.

The opposite of $^{-}282$ is $^{+}282$, so we get: $^{+}20,320$ - $^{-}282$ = $^{+}20,320$ + $^{+}282$ = $^{+}20,602$

In the above problem, we added the opposite of the second integer and subtraction was transformed into addition. Let's look at some simpler examples of subtracting integers.

Example 1: $^{+}5$ - $^{+}2$



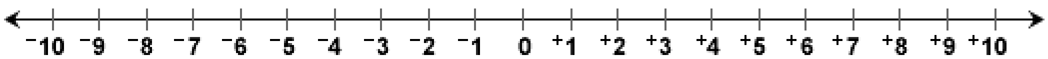
Step 1: The opposite of ${}^{+}2$ is ${}^{-}2$.

Step 2: Subtraction becomes addition.

Solution: ${}^{+}5 - {}^{+}2 = {}^{+}5 + {}^{-}2 = {}^{+}3$

Example 2:

Find the difference between each pair of integers.

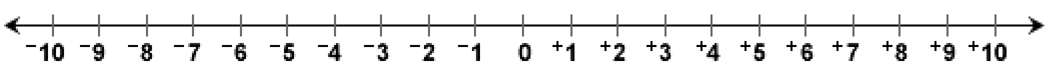


Subtracting Integers		
Subtract	**Add The Opposite**	**Result**
${}^{+}9 - {}^{+}4 =$	${}^{+}9 + {}^{-}4 =$	${}^{+}5$
${}^{+}9 - {}^{-}4 =$	${}^{+}9 + {}^{+}4 =$	${}^{+}13$
${}^{-}9 - {}^{+}4 =$	${}^{-}9 + {}^{-}4 =$	${}^{-}13$
${}^{-}9 - {}^{-}4 =$	${}^{-}9 + {}^{+}4 =$	${}^{-}5$

Notice that in each problem above, the first integer remained unchanged. Also, do not confuse the sign of the integer with the operation being performed. Remember that:
${}^{-}9 + {}^{+}4 = {}^{-}5$ is read as *Negative 9 plus positive 4 equals negative 5*.

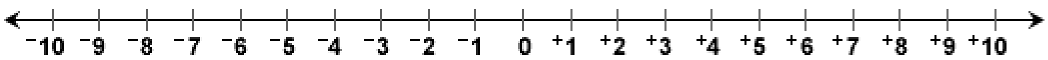
Let's look at some more examples:

Example 3: Find the difference between each pair of integers.



Subtracting Integers		
Subtract	**Add The Opposite**	**Result**
${}^{+}7 - {}^{+}10 =$	${}^{+}7 + {}^{-}10 =$	${}^{-}3$
${}^{+}7 - {}^{-}10 =$	${}^{+}7 + {}^{+}10 =$	${}^{+}17$
${}^{-}7 - {}^{+}10 =$	${}^{-}7 + {}^{-}10 =$	${}^{-}17$
${}^{-}7 - {}^{-}10 =$	${}^{-}7 + {}^{+}10 =$	${}^{+}3$

Example 4: Find the difference between each pair of integers. You may extend the number line below to help you solve these problems.



Subtracting Integers		
Subtract	**Add The Opposite**	**Result**
$^{-}8 - {}^{+}3 =$	$^{-}8 + {}^{-}3 =$	$^{-}11$
$^{+}17 - {}^{-}9 =$	$^{+}17 + {}^{+}9 =$	$^{+}26$
$^{-}12 - {}^{+}15 =$	$^{-}12 + {}^{-}15 =$	$^{-}27$
$^{-}19 - {}^{-}23 =$	$^{-}19 + {}^{+}23 =$	$^{+}4$

Summary: To subtract an integer, add its opposite. When subtracting integers, it is important to show all work, as we did in this lesson. If you skip steps, or do the work in your head, you are very likely to make a mistake--even if you are a top math student!

Practice Adding and Subtracting Integers

Perform the operations

1) $-12 + 8$
2) $-15 + 12$
3) $-6 + (-2)$
4) $-18 + (-6)$
5) $15 + (-9)$
6) $13 + (-17)$
7) $9 + (-2)$
8) $10 + (-6)$
9) $-18 + 12$
10) $-12 + 15$
11) $-8 + 10$
12) $-9 + 1$
13) $-4 + 3 + (-5)$
14) $2 + (-9) + 5$
15) $-4 + (-7) + 3$
16) $2 + (-2)$
17) $4 + (-7) + 3$
18) $-1 + (-7) + (-9)$
19) $2 - 8$
20) $7 - 10$
21) $-7 - 12$
22) $-15 - 12$
23) $16 - 10$
24) $8 - 13$
25) $-9 - 10$
26) $-3 - 12$
27) $5 - (-3)$
28) $8 - (-3)$
29) $-9 - (-8)$
30) $-13 - (-5)$
31) $-12 + 9$
32) $-8 + 8$
33) $-10 - 15$
34) $-13 - (-13)$
35) $5 - (-6) - (-3)$
36) $-12 - 14$
37) $-15 - (-8) + 2$
38) $8 - 4 - 2$
39) $-10 - 6 - (-7)$
40) $-4 - (-1) + 7$

Answers

1) -4
2) -3
3) -8
4) -24
5) 6
6) -4
7) 7
8) 4
9) -6
10) 3
11) 2
12) -8
13) -6
14) -2
15) -8
16) 0
17) 0
18) -17
19) -6
20) -3
21) -19
22) -27
23) 6
24) -5
25) -19
26) -15
27) 8
28) 11
29) -1
30) -8
31) -3
32) 0
33) -25
34) 0
35) 14
36) -26
37) -5
38) 2
39) -9
40) 4

Multiplying and Dividing Integers

Perform the following operations

1) $5(-8)$
2) $-5(9)$
3) $-12(-6)$
4) $\frac{-18}{2}$
5) $\frac{-36}{-9}$
6) $\frac{50}{-10}$
7) $-9(-7)$
8) $-5(-3)(-2)$
9) $(-7)^2$
10) $(-5)^3$
11) $-15(0)$
12) $\frac{0}{-8}$
13) $\frac{25}{0}$
14) $\frac{-16}{-2}$
15) $\frac{28}{-4}$
16) $8(-5)(-9)$
17) -2^4
18) $-3(5)(-4)$
19) $\frac{-6(-4)}{-2}$
20) $\frac{4(-8)}{-2}$
21) -2^2
22) $(-3)^3$
23) $3(-2)(3)$

Evaluate the following expressions

24) xy for $x = 3$ and $y = -4$
25) xy for $x = -5$ and $y = -6$
26) xy for $x = 8$ and $y = -3$
27) $\frac{x}{y}$ for $x = -16$ and $y = 4$
28) $\frac{x}{y}$ for $x = 6$ and $y = -3$
29) $\frac{x}{y}$ for $x = -12$ and $y = -4$
30) $\frac{x}{y}$ for $x = 15$ and $y = -5$

Answers

1) -40
2) -45
3) 72
4) -9
5) 4
6) -5
7) 63
8) -30
9) 49
10) -125
11) 0
12) 0
13) $undefined$
14) 8
15) -7
16) 360
17) -16
18) 60
19) -12
20) 16
21) -4
22) -27
23) -18
24) -12
25) 30
26) -24
27) -4
28) -2
29) 3
30) -3

Exponents, Square Roots, and Order of Operations

Evaluate

1) 7^3

2) 6^2

3) 3^3

4) 25^1

5) $5^1 \cdot 2^2$

6) $7 \cdot 2^3$

7) 9^2

8) $3^2 \cdot 2^3$

9) $4^1 \cdot 3^2$

10) $5^1 \cdot 2^3$

Find the Square Root

11) $\sqrt{4}$

12) $\sqrt{25}$

13) $\sqrt{36}$

14) $\sqrt{0}$

15) $\sqrt{100}$

16) $\sqrt{144}$

17) $\sqrt{196}$

18) $\sqrt{64}$

19) $\sqrt{49}$

Simplify

20) $3 \cdot 5 + 3 \cdot 6 + 7$

21) $3 + 8(10)$

22) $2^2 \cdot 3^2 + (33 - 19) \cdot 3$

23) $2(3 + 1)^2$

24) $4 \cdot 6 + 8(10 + 4) + 4$

25) $54 \div 9 \cdot (14 - 9)$

26) $9(5 - 1) + \sqrt{9}$

27) $15 \cdot \sqrt{64} - 10 \cdot \sqrt{64}$

28) $7(3)^2$

29) $7 + 3 - 4 + 6$

30) $7 + 3 - (4 + 6)$

Basic Equations

Solve for x

1. $3x + 2 = x - 6$
2. $5x - 8 = 3x + 4$
3. $8x - 6 = 2x - 24$
4. $4x + 2 = x - 10$
5. $-3x - 8 = 2x + 7$
6. $-5x + 1 = 3x - 7$
7. $6x - 4 = -2x - 20$
8. $-3x + 2 = -5x + 10$
9. $-10x + 1 = -4x + 19$
10. $-7x - 4 = -3x - 16$
11. $3x + 9 - 2x - 8 = 6x - 24$
12. $4x - 3 - 8x + 1 = -5x + 9$
13. $10(-2x + 1) = -19(x + 1)$
14. $2(2 - 3x) = -5(x - 3)$
15. $4(2x - 3) + 7 = x + 5 + 2x$
16. $3(5x - 2) - 2 = 15x + 4 - 2x$

ANSWERS

1. $x = -4$
2. $x = 6$
3. $x = -3$
4. $x = -4$
5. $x = -3$
6. $x = 1$
7. $x = -2$
8. $x = 4$
9. $x = -3$
10. $x = 3$
11. $x = 5$
12. $x = 11$
13. $x = 29$
14. $x = -11$
15. $x = 2$
16. $x = 6$

Using the FOIL Method

You already know how to simplify an expression like 7(4x + 3), right? Just use the distributive property to multiply 7 times 4x and 7 times 3. This gives you an answer of 28x + 21. Pretty simple. But - What if you have something like this: (4x + 6)(x + 2)? That's where we use the FOIL method. FOIL means first, outside, inside, last. That's not too hard to remember if you say it in your head a few times.

You use FOIL to multiply the terms inside the parenthesis in a specific order: first, outside, inside, last. Here's how to solve (4x + 6)(x + 2):

First - multiply the first term in each set of parenthesis: $4x * x = 4x^2$

$$(4x + 6)(x + 2)$$

Outside - multiply the two terms on the outside: 4x * 2 = 8x

$$(4x + 6)(x + 2)$$

Inside - multiply both of the inside terms: 6 * x = 6x

$$(4x + 6)(x + 2)$$

Last - multiply the last term in each set of parenthesis: 6 * 2 = 12

$$(4x + 6)(x + 2)$$

Now just add everything together to get $4x^2 + 14x + 12$. This method only works easily with two binomials. To multiply something complicated like (4x + 6)(5x - 3)(15 - x), just do FOIL on two of the binomials and then distribute the answer onto the remaining binomial.

Here are some more examples of FOIL multiplication:

Example:

Multiply the following: (6x+1)(2x+9)

Solution:

Just follow the letters in FOIL:
First: $6x*2x=12x^2$.
Outside: $6x*9=54x$.
Inside: $1*2x=2x$.
Last: $1*9=9$.

Sum it all up and you get: $(12x^2+56x+9)$.

Example:

Multiply the following: (2x-5)(x-4)

Solution:

Just follow the letters in FOIL:
First: $2x*x=2x^2$.
Outside: $2x*(-4)=-8x$.
Inside: $-5*x=-5x$.
Last: $(-5)*(-4)=20$.

Sum it all up and you get: $(2x^2-13x+20)$.

The FOIL method is not too difficult to learn once you remember what it stands for. Just repeat first, outside, inside, last and you'll remember it. Other than that, it's just a matter of multiplying each of those steps and adding everything together. Even if the numbers are really ugly with fractions and negative signs and such just follow the steps and the method will work.