## Activities/ <br> Resources <br> for <br> Unit IV: Percents

## What is a Percent?

## A percent is a ratio of a number to $\mathbf{1 0 0}$. A percent can be expressed using the percent symbol \%.

Example: 10 percent or $10 \%$ are both the same, and stand for the ratio 10:100.

## Percent as a fraction

A percent is equivalent to a fraction with denominator 100.
Example: $5 \%$ of something $=5 / 100$ of that thing.
Example: $21 / 2 \%$ is equal to what fraction?
Answer:
$21 / 2 \%=(21 / 2) / 100=5 / 200=1 / 40$
Example: $52 \%$ most nearly equals which one of $1 / 2,1 / 4,2,8$, or $1 / 5$ ?
Answer: $52 \%=52 / 100$. This is very close to $50 / 100$, or $1 / 2$.

## Percent as a decimal

Percent and hundredths are basically equivalent. This makes conversion between percent and decimals very easy.

To convert from a decimal to a percent, just move the decimal 2 places to the right. For example, $0.15=15$ hundredths $=15 \%$.

Example:
$0.0006=0.06 \%$
Converting from percent to decimal form is similar, only you move the decimal point 2 places to the left. You must also be sure, before doing this, that the percentage itself is expressed in decimal form, without fractions.

## Example:

Express $3 \%$ in decimal form. Moving the decimal 2 to the left (and adding in 0 's to the left of the 3 as place holders), we get 0.03 .

## Example:

Express $971 / 4 \%$ in decimal form. First we write $971 / 4$ in decimal form: 97.25 . Then we move the decimal 2 places to the left to get 0.9725 , so $971 / 4 \%=0.9725$. This makes sense, since $971 / 4 \%$ is nearly $100 \%$, and 0.9725 is nearly 1 .

## Estimating percents

When estimating percents, it is helpful to remember the fractional equivalent of some simple percents.
$100 \%=1$
(100\% of any number equals that number.)
$50 \%=1 / 2=0.5$
( $50 \%$ of any number equals half of that number.)
$25 \%=1 / 4=0.25$
( $25 \%$ of any number equals one-fourth of that number.)
$10 \%=1 / 10=0.1$
( $10 \%$ of any number equals one-tenth of that number.)
$1 \%=1 / 100=0.01$
( $1 \%$ of any number equals one-hundredth of that number.)
Because it is very easy to switch between a decimal and a percent, estimating a percent is as easy as estimating a fraction as a decimal, and converting to a percent by multiplying by 100 .

## Example:

Estimate 19 as a percent of 80 .
As a fraction, $19 / 80 \cong 20 / 80=1 / 4=0.25=25 \%$. The step used to estimate the percent occurred when we estimated $19 / 80$ as 20/80.
The exact percent is actually $23.75 \%$, so the estimate of $25 \%$ is only $1.25 \%$ off. (About 1 part in 100.)

## Example:

## Estimate 7 as a percent of 960 .

As a fraction, $7 / 960 \cong 7 / 100=0.007=0.7 \%$. The step used to estimate the percent occurred when we estimated $7 / 960$ as $7 / 1000$.
The exact percent, to the nearest thousandth of a percent, is actually $0.729 \%$.
To estimate the percent of a number, we may convert the percent to a fraction, if useful, to estimate the percent.

## Example:

## Estimate $13 \%$ of 72.

Twice $13 \%$ is $26 \%$, which is very close to $25 \%$, and $25 \%=1 / 4$. We may multiply both sides by $1 / 2$ to get an estimate for $13 \%$ : $13 \% \cong 12.5 \%=1 / 2 \times 25 \%=1 / 2 \times 1 / 4=1 / 8$. Using our estimate of $1 / 8$ for $13 \%, 1 / 8 \times 72=9$, so we get an estimate of 9 for $13 \%$ of 72.

If we had calculated this exactly, $13 \%$ of 72 equals 9.36 . It may look like we did a lot more work to get the estimate of 9 that just multiplying 72 by 0.13 , but with practice, keeping in mind some simple percents and the fractions they are equal to will enable you to estimate some number combinations very quickly.

## Interest

Interest is a fee paid to borrow money. It is usually charged as a percent of the total amount borrowed. The percent charged is called the interest rate. The amount of money borrowed is called the principal. There are two types of interest, simple interest and compound interest.

Example: A bank charges $7 \%$ interest on a $\$ 1000$ loan. It will cost the borrower $7 \%$ of $\$ 1000$, which is $\$ 70$, for each year the money is borrowed. Note that when the loan is up, the borrower must pay back the original $\$ 1000$.

## Simple Interest

Simple interest is interest figured on the principal only, for the duration of the loan. Figure the interest on the loan for one year, and multiply this amount by the number of years the money is borrowed for.

Example: A bank charges $8 \%$ simple interest on a $\$ 600$ loan, which is to be paid back in two years. It will cost the borrower $8 \%$ of $\$ 600$, which is $\$ 48$, for each year the money is borrowed. Since it is borrowed for two years, the total charge for borrowing the money will be $\$ 96$. After the two years the borrower will still have to pay back the original $\$ 600$.

## Compound Interest

Compound interest is interest figured on the principal and any interest owed from previous years. The interest charged the first year is just the interest rate times the amount of the loan. The interest charged the second year is the interest rate, times the sum of the loan and the interest from the first year. The interest charged the third year is the interest rate, times the sum of the loan and the first two years' interest amounts. Continue figuring the interest in this way for any additional years of the loan.

Example: A bank charges $8 \%$ compound interest on a $\$ 600$ loan, which is to be paid back in two years. It will cost the borrower $8 \%$ of $\$ 600$ the first year, which is $\$ 48$. The second year, it will cost $8 \%$ of $\$ 600+\$ 48=\$ 648$, which is $\$ 51.84$. The total amount of
interest owed after the two years is $\$ 48+\$ 51.84=\$ 99.84$. Note that this is more than the $\$ 96$ that would be owed if the bank was charging simple interest.

Example: A bank charges 4\% compound interest on a $\$ 1000$ loan, which is to be paid back in three years. It will cost the borrower 4\% of $\$ 1000$ the first year, which is $\$ 40$. The second year, it will cost $4 \%$ of $\$ 1000+\$ 40=\$ 1040$, which is $\$ 41.60$. The third year, it will cost $4 \%$ of $\$ 1040+\$ 41.60=\$ 1081.60$, which is $\$ 43.26$ (with rounding). The total amount of interest owed after the three years is $\$ 40+\$ 41.60+43.26=\$ 124.86$.

## Percent increase and decrease

Percent increase and decrease of a value measure how that value changes, as a percentage of its original value.

Example: A collectors' comic book is worth $\$ 120$ in 1994, and in 1995 its value is $\$ 132$.
The change is $\$ 132-\$ 120=\$ 12$, an increase in price of $\$ 12$; since $\$ 12$ is $10 \%$ of $\$ 120$, we say its value increased by 10\% from 1994 to 1995.

Example: A bakery makes a chocolate cake that has 8 grams of fat per slice. A new change in the recipe lowers the fat to 6 grams of fat per slice. The change is 8 g $6 \mathrm{~g}=2 \mathrm{~g}$, a decrease of 2 grams; since 2 grams is $25 \%$ of 8 , we say that the new cake recipe has $25 \%$ less fat, or a $25 \%$ decrease in fat.

Example: Amy is training for the 1500 meter run. When she started training she could run 1500 meters in 5 minutes and 50 seconds. After a year of practice her time decreased by $8 \%$. How fast can she run the race now? Her old time was $5 \times 60+$ $50=350$ seconds, and $8 \%$ of 350 is 28 , so she can run the race in $350-28=322$ seconds (5 minutes and 22 seconds).

## Percent Discount

A discount is a decrease in price, so percent discount is the percent decrease in price.
Example: Chocolate bars normally cost 80 cents each, but are on sale for 40 cents each, which is $50 \%$ of 80 , so the chocolate is on sale at a $50 \%$ discount.

Example: A compact disc that sells for $\$ 12$ is on sale at a $20 \%$ discount. How much does the disc cost on sale? The amount of the discount is $20 \%$ of $\$ 12$, which is $\$ 2.40$, so the sale price is $\$ 12.00-\$ 2.40=\$ 9.60$.

Example: Movie tickets sell for $\$ 8.00$ each, but if you buy 4 or more you get $\$ 1.00$ off each ticket. What percent discount is this? We figure $\$ 1$ as a percentage of $\$ 8$ : $\$ 1.00 / \$ 8.00 \times 100 \%=12.5 \%$, so this is a $12.5 \%$ discount.

## Simple Interest Formula

```
Calculating Interest: Principal, Rate and Time are Known
$4500.00 at 9.5% for 6 years
I = Prt
I = (4500.00) (0.095) (6)
I = $2565.00
$8700.00 at 3.25% for 3 years
I = Prt
I = (8700.00) (0.0325)(3)
I =$848.25
```

```
$6300.00 at 8% for 310 years
I = Prt
I = (6300.00)(0.08)(310
I = $428.05
```

Let's say you want to borrow \$6300.00 from March 15th, 2011 until January 20th 2012 at a rate of $8 \%$. The formula will still be I = Prt, however, you need to calculate the days. To do so, you will not count the day the money is borrowed or the day the money is returned. Let's figure out the days: March = 16, April = 30, May = 31, June = 30, July = 31, August $=31$, September $=30$, October $=31$, November $=30$, December $=31$, January $=19$. Therefore the time is $310 / 365$. A total of 310 days out of 365 . This is entered into the $t$ for the formula.

